Online Optimization over RIS Networks via Mixed Integer Programming

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Joint work with

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Preliminary work[Kajihara+ 25] appeared at ICMU2025

Online Optimization in Communication Engineering(CE)

- Many of recent work of CE uses machine learning techniques including deep learning

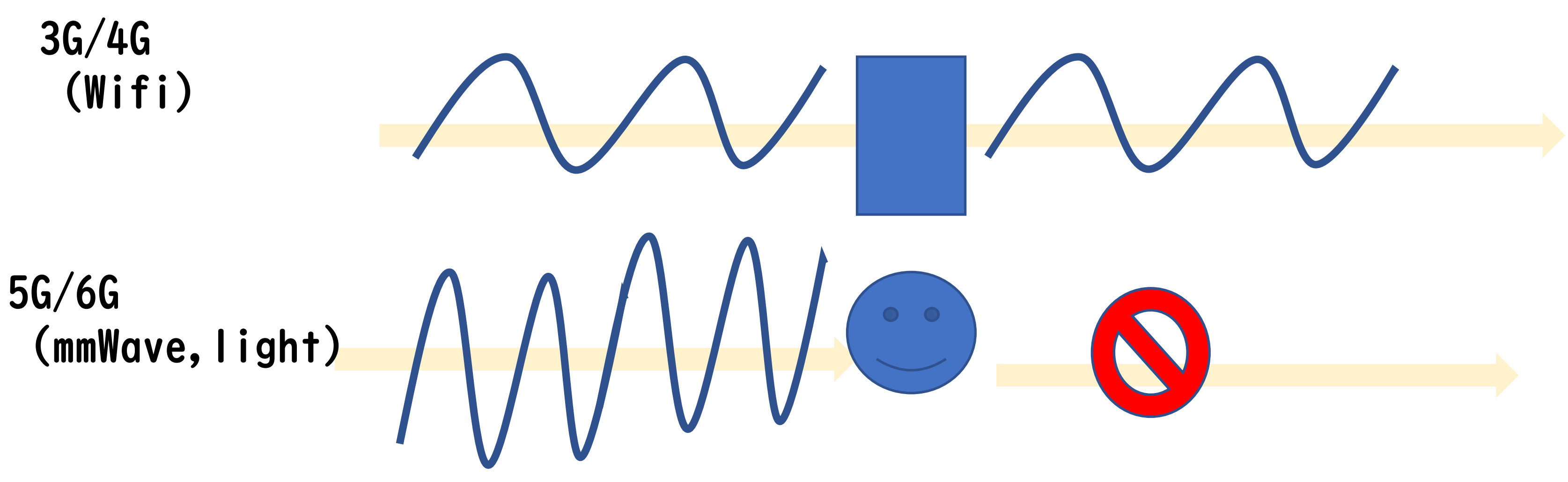
- Some work focuses on online optimization in CE with online learning, e.g., bandit
  - Channel selection
    (channels are arms)
  - Online optimization in networks using UAVs
    (places are arms)

- Online Optimization techniques can contribute to CE
  - Many tasks need to be done online
  - Robustness to changing environments

Background

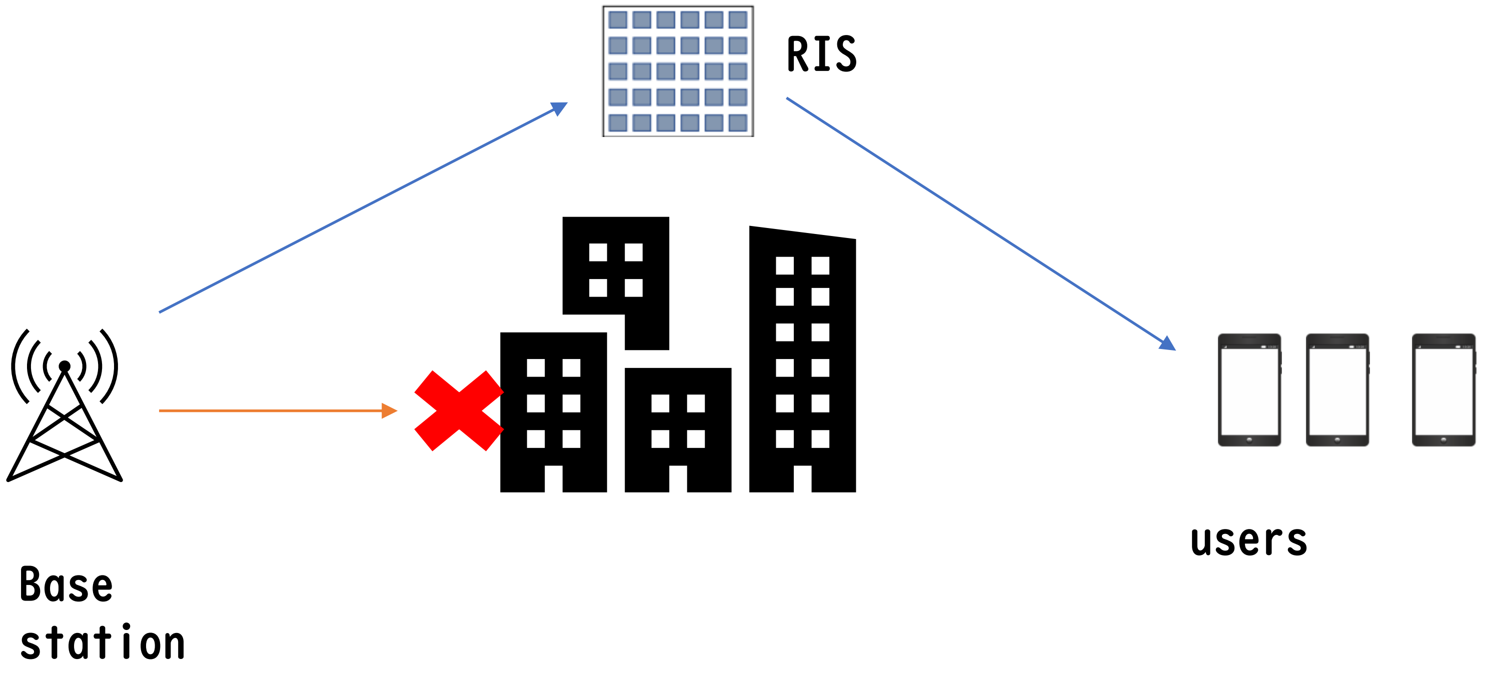
- Necessity of Optimization in 5G/6G wireless communication
  - Infrastructure with higher throughput
  - 5G/6G systems use higher frequency waves than Wifi
  - Vulnerable to obstacles

    selective communication necessary

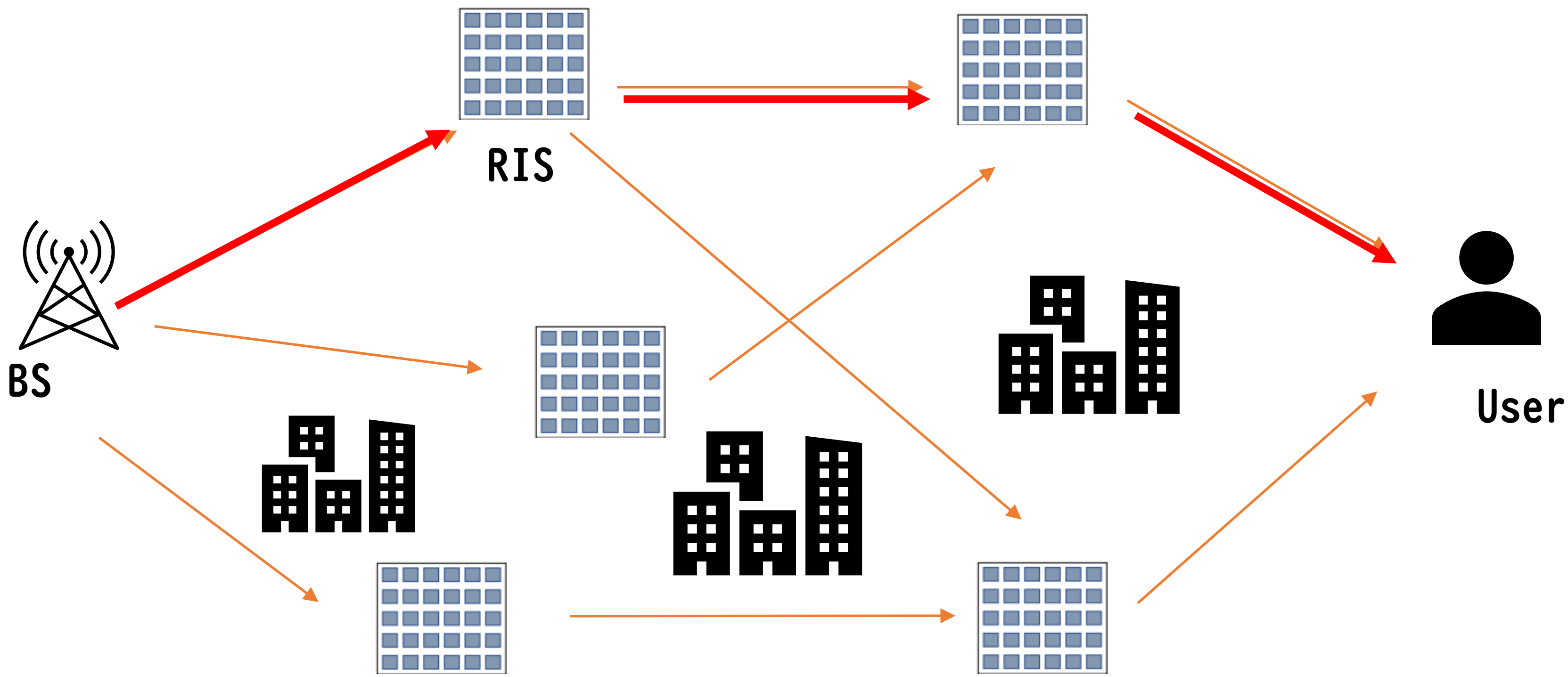


Reconfigurable Intelligent Surface (RIS, aka IRS)

- Consists of multiple reflective units
- Each reflective unit can reflect signals and control their angles and strength



Network of RIS [Cf. Asif+20]



Advantage: can avoid obstacles
Disadvantage: Relaying with RIS weaken the strength of waves due to distance, weather, moving obstacles and etc.

Want to find a path optimizing the strength of received signal to user

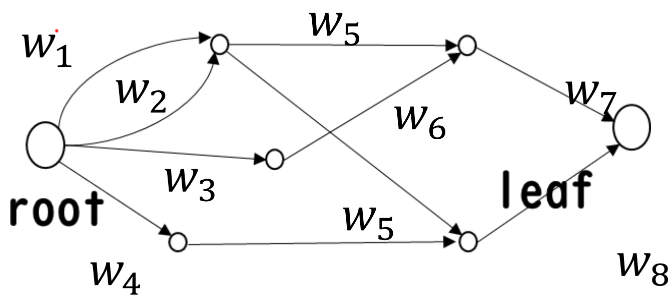
Related Work [Asif+ 22]

Input: DAG $G = (V, E)$ with single root and leaf,
decreasing ratios $\boldsymbol{w} \in [0,1]^E$

Output: $P^* = \underset{P \in \mathcal{P}}{\operatorname{argmax}} \prod_{e \in P} w_t(e)$

path maximizing product of weight along with it

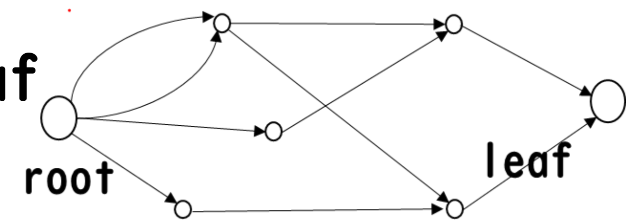
$\mathcal{P}$: set of paths from root to leaf



- Offline problem solved by Dijkstra method
- Online setting not considered

Our Formulation
(Adversarial full info. setting)

Given: DAG $G = (V, E)$ with root and leaf

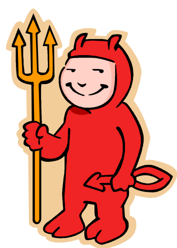


For each round $t = 1,2, \ldots, T$:

Player

1. Path $P_t \in \mathcal{P}$

Adversary



2. Decreasing ratios $\boldsymbol{w_t} \in [0,1]^E$

3. Received power $g_t(P_t) = \prod_{e \in P_t} w_t(e)$

Formulation(2)

Goal：Minimize

$$\text{Regret}(\mathrm{T}) = \max_{P^* \in \mathcal{P}} \sum_{t=1}^{T} g_t(P^*) \quad - \quad \sum_{t=1}^{T} g_t(P_t)$$

$$= \max_{P^* \in \mathcal{P}} \sum_{t=1}^{T} \prod_{e \in P^*} w_t(e) - \sum_{t=1}^{T} \prod_{e \in P_t} w_t(e)$$

Cumulative received power of best fixed path

Cumulative received power of Player

- Combinatorial Online Prediction with non-linear rewards
- Different from standard Online shortest (longest) path problem with additive rewards

Our Contribution

- Formulation of Online Optimization over RIS network

- We show
  - Corresponding offline problem is NP-hard
  - But reformulated as a Mixed Integer Program(MIP)

- Implication
  - Low Regret Guarantee by FPL[Suggala+ 20] using a MIP solver

Offline Problem:

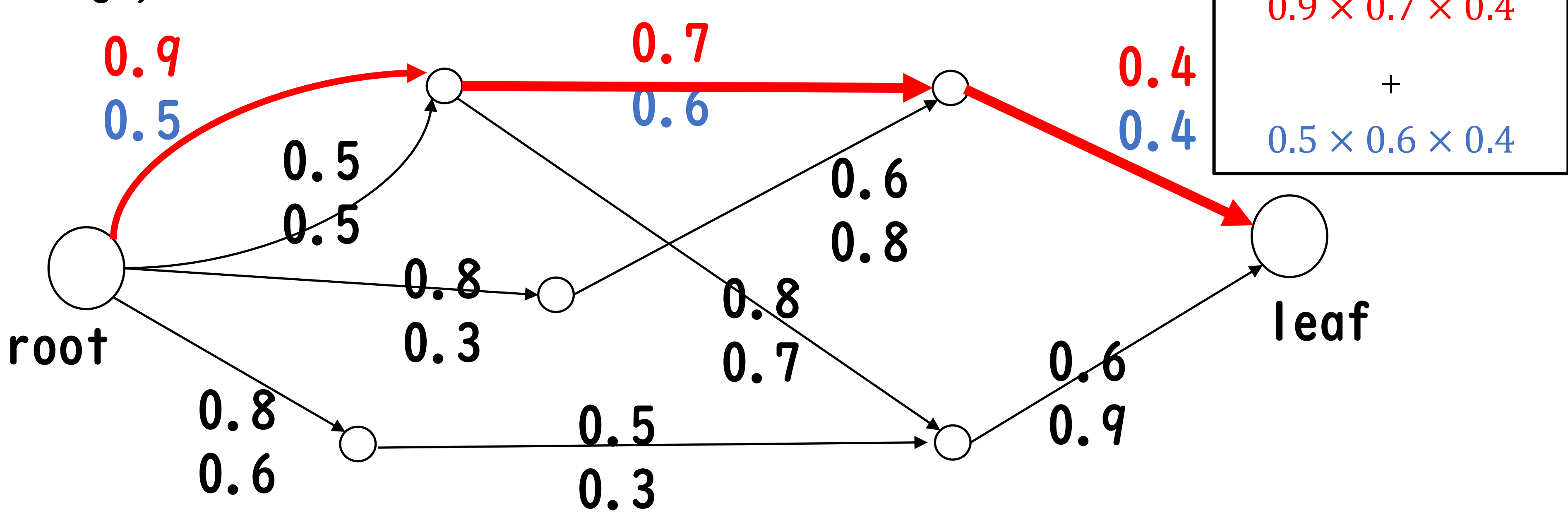
Sum of Product Longest Path Problem (SPLP)

Input: DAG $G = (V, E)$,

decreasing ratios $\boldsymbol{w}_1, \boldsymbol{w}_2, \dots, \boldsymbol{w}_T$ $(\boldsymbol{w}_t \in [0,1]^E)$

Output: $P^* = \underset{P \in \mathcal{P}}{\operatorname{argmax}} \sum_{t=1}^{T} \prod_{e \in P} w_t(e)$

E.g., $T = 2$



NP-hardness of SPLP

- Reduction from 2-MinSAT[Kohli+,94]

2-MinSAT

Input:2-CNF(CNF with at most 2 literals in each clause)

E.g. $(x_1 \vee \overline{x_3}) \wedge x_4 \wedge (\overline{x_1} \vee x_5)$

Output: Is there an assignment $a \in \{0,1\}^n$ for which $\leq k$ clauses satisfied?

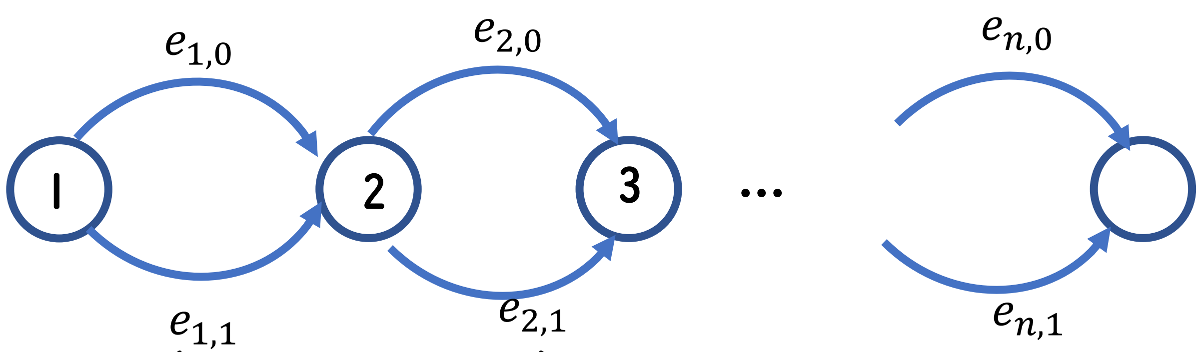
E.g. yes for $k = 1$ with $a = 01100$

Thm. [Kohli+,94] 2-MinSAT is NP-complete

NP-hardness of SPLP(2): Reduction from 2-MinSAT to SPLP

Input:2-CNF $C_1 \wedge C_2 \wedge \cdots \wedge C_T$

Construct the DAG G=(V,E)



Each assignment $\boldsymbol{a}$ corresponds to Path P_a

Weights (for $t = 1, \ldots, T$)

$$w_t(e_{i,0}) = \begin{cases} 0, & if\ \overline{x_i} \in C_t \\ 1, & o.w. \end{cases} \qquad w_t(e_{i,1}) = \begin{cases} 0, & if\ x_i \in C_t \\ 1, & o.w. \end{cases}$$

Prop. $\prod_{e \in P_a} w_t(e) = 0\ (= 1) \Leftrightarrow C_t(\boldsymbol{a}) = 1 (= 0)$

NP-hardness of SPLP(3)

Prop. $\prod_{e \in P_{\boldsymbol{a}}} w_t(e) = 0 \ (= 1) \Leftrightarrow C_t(\boldsymbol{a}) = 1 (= 0)$

Implies

$$T - \sum_{t=1}^{T} \prod_{e \in P_{\boldsymbol{a}}} w_t(e) = \sum_{t=1}^{T} C_t(\boldsymbol{a})$$

So, $P_{a^*} = \arg\max_{P \in \mathcal{P}} \Leftrightarrow \boldsymbol{a}^* = \arg\min_{\boldsymbol{a} \in \{0,1\}^n} \sum_{t=1}^{T} C_t(\boldsymbol{a})$

Poly-time algorithm for SPLP implies one for 2-MinSAT

MIP for SPLP

Initial Formulation

$$\max_{P \in \{0,1\}^E} \sum_{t=1}^{T} \prod_{e \in P} w_t(e) \left(= \sum_{t=1}^{T} \prod_{e \in E} w_t(e)^{P_e} \right)$$

sub. to: P is a path

Convex non-linear optimization at a glance

Flow Constraints

$P \in \{0,1\}^E$ is a path

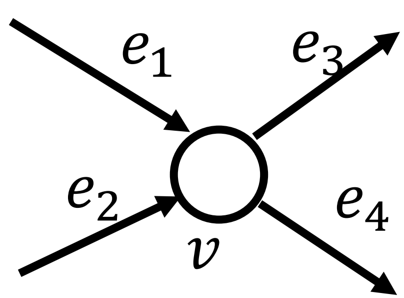
$\Leftrightarrow$

$\sum_{e.u=root} P_e = 1$

$\sum_{e.v=v} P_e = \sum_{e.u=v} P_e$ for any $v \in V \setminus \{root\}$

(flow constraints)

where edge e starts from node e.u towards node e.v



$P_{e_1} + P_{e_2} = P_{e_3} + P_{e_4} = 1$

Received Power Constraints

New variables

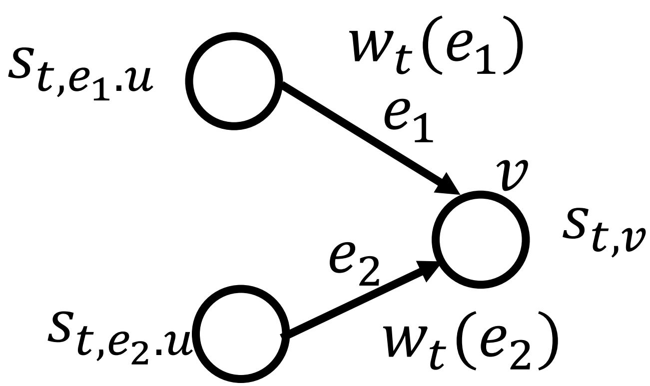
$s_{t,v}$: sum of received power on node v at round t

Then, objective is $\sum_{t=1}^{T} \prod_{e \in P^*} w_t(e) = \sum_{t=1}^{T} s_{t,leaf}$ (linear),

provided that

$\sum_{e.v=v} w_t(e) P_e s_{t,e.u} = s_{t,v}$ for $v \in V \setminus \{root\}$ (bilinear)

Received power constraints



$w_t(e_1) P_{e_1} s_{t,e_1.u} + w_t(e_2) P_{e_2} s_{t,e_2.u}$
$= s_{t,v}$

Received Power Constraints(2)

New variable $z_{t,e}$
s.t.

$$z_{t,e} = P_e s_{t,e.u}$$
$$\Leftrightarrow$$
$$0 \leq z_{t,e} \leq P_e \quad (P_e = 0 \Rightarrow z_{t,e} = 0)$$
$$-(1 - P_e) \leq z_{t,e} - s_{t,e.u} \leq 0 \quad (P_e = 1 \Rightarrow z_{t,e} = s_{t,e.u})$$

NOTE: This does not hold for relaxed $P_e \in [0,1]$

Then,

$$\sum_{e.v=v} w_t(e) P_e s_{t,e.u} = s_{t,v} \text{ for } v \in V \setminus \{root\} \quad \text{(bilinear)}$$
$$\Leftrightarrow$$
$$\sum_{e.v=v} w_t(e) z_t = s_{t,v}$$
$$0 \leq z_t \leq P_e$$
$$-(1 - P_e) \leq z_t \leq s_{t,e.u} \quad \text{(linear)}$$

Thm. SPLP is a MIP

$$\arg\max_{\boldsymbol{P},\boldsymbol{s},\boldsymbol{z}} \sum_{t=1}^{T} s_{t,leaf}$$

sub. to

$$\sum_{e:e.u=root} P_e = 1,$$

$$\sum_{e:e.u=v} P_e = \sum_{e:e.v=v} P_e, \forall v \in V \setminus \{root\},$$

Flow Constraints

$$s_{t,root} = 1,$$

$$\sum_{e:e.v=v} w_t(e) z_{t,e} = s_{t,v}, \forall v \in V \setminus \{root\}, \forall t \in [T],$$

$$0 \leq z_{t,e} \leq P_e,\ \forall e \in E\ ,\ \forall t \in [T],$$

$$P_e - 1 \leq z_{t,e} - s_{t,e.u} \leq 0\ \forall e \in E\ ,\ \forall t \in [T],$$

$$\boldsymbol{P} \in \{0,1\}^E, \boldsymbol{s} \in [0,1]^{V \times T}, \boldsymbol{z} \in [0,1]^{E \times T}$$

Received Power Constraints

$|E| + |V|T + |E|T$ Variables $|V| + |E| + T + |V|T + 4|E|T$ Constraints

Implications to Online Problem

- Online Problem can be solved using FPL[Sugalla+ 20] for smooth non-linear reward functions

- Prop. $g_t(P)$ is 1-Lipschitz w.r.t. 1-norm
  - $|g_t(P) - g_t(P')| \leq 1 \leq \| P - P' \|_1$ for any $P, P' \in \mathcal{P}$

  reward function is smooth

<u>FPL</u> $P_t \in \mathrm{argmax}_{P \in \mathcal{P}} \{\sum_{\tau < t} g_\tau(P) + \sigma_t \cdot P\}$,

where $\sigma_t \sim \left(\mathrm{Exp}(\eta)\right)^E$

MIP

Follow the Perturbed Leader Implications to Online Problem

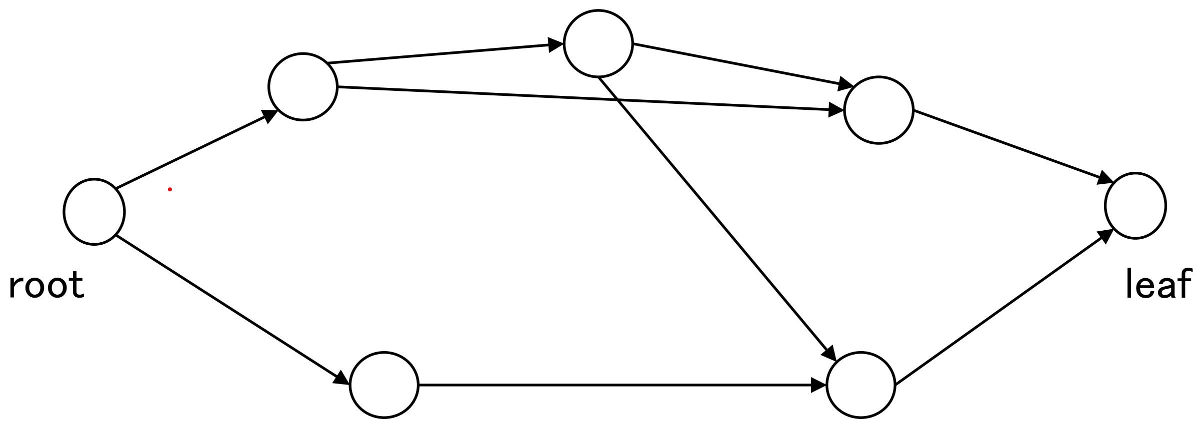
Coro. [Suggala+, 20]

FPL achieves

$$Regret(T) = O\left(|E|^{2.5}\sqrt{T}\right),$$

Experiments

- MIP solver : Gurobi optimizer 11.0.3
- Algorithms : FPL[Suggala+, 20] & FTL (without perturbation)
- DAG $G = (V, E)$ (Same as [Asif+, 22])

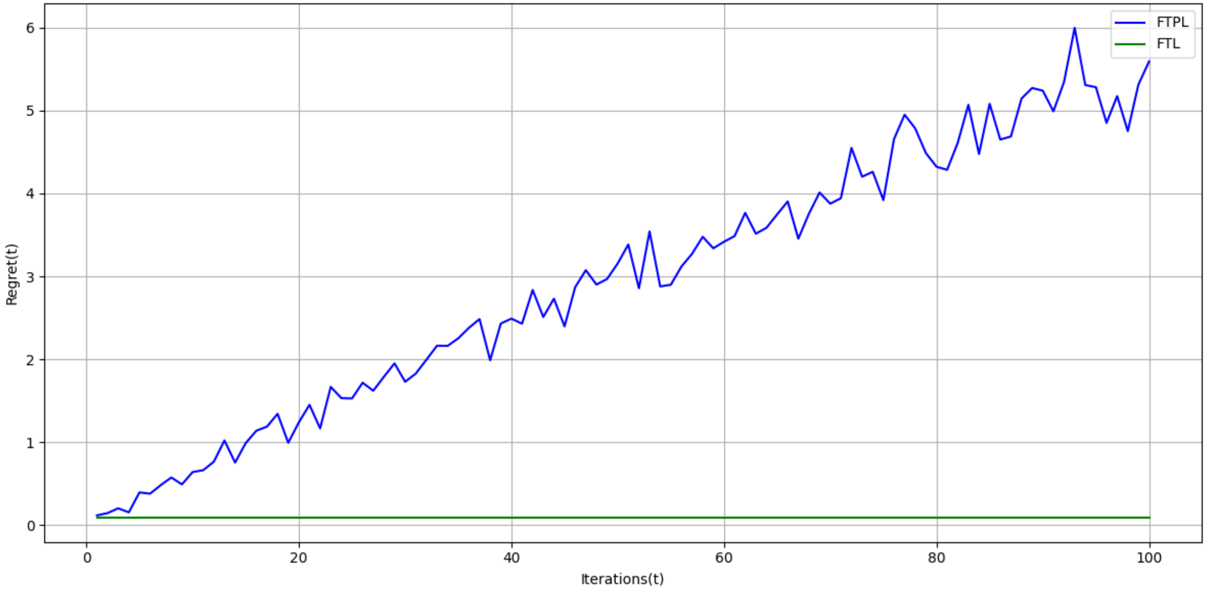


- $T = 100$

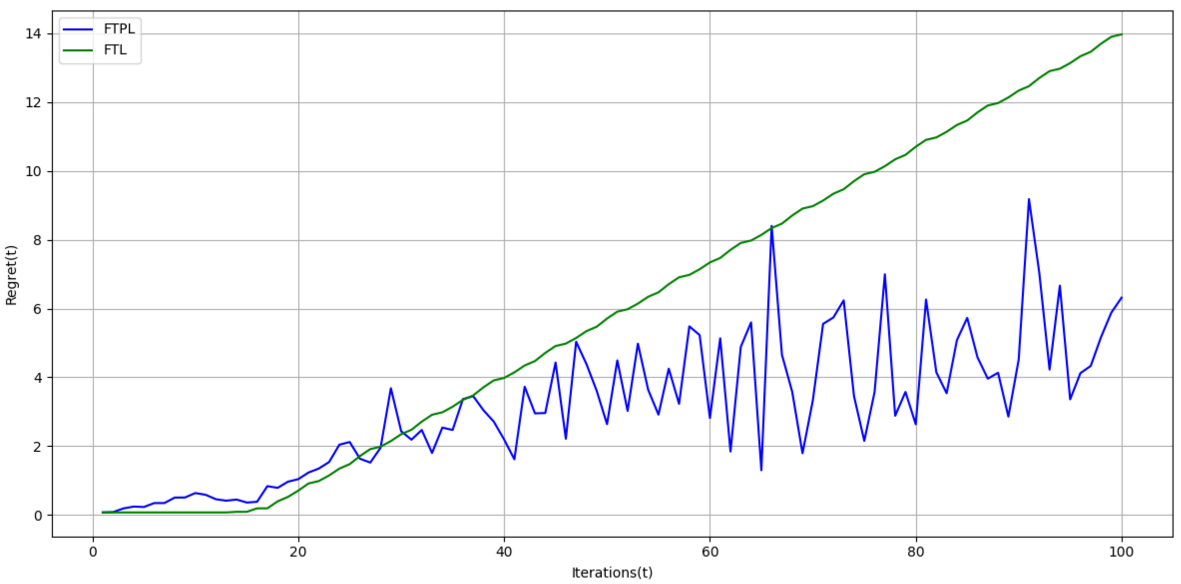
[Ex 1] $w_t(e)$ iid from uniform distribution with different means

[Ex 2] adversarial to FTL

Ex 1:random weights



Ex 2：adversarial weights



Summary

- Combinatorial Online Optimization over RIS network
  - Offline problem is NP-hard but MIP
  - FPL can be used to obtain low regret

- Open Questions
  - Regret bound for stochastic environments
  - Bandit Extensions
  - Efficient approximation algorithms
    Cf. 2-MinSAT has 2-approximation algorithm
  - Can MIP property used to obtain better bounds?

References

- [Asif+ 22] A. B. Asif et al., "Optimal path selection in cascaded intelligent reflecting surfaces," IEEE VTC2022-Fall, 1–5, 2022.
- [Suggala+ 20] A. S. Suggala, P. Netrapalli, "Online Non-Convex Learning: Following the Perturbed Leader is Optimal," ALT2020, PMLR 117:845-861, 2020.
- [Kajihara+ 25]S.Kajihara, et al., "Online Path Optimization in Cascaded RIS Networks via Mixed Integer Programming," ICMU2025, 2025.