

EVALUATION
ONLINE LEARNING
LINKS WITH OPTIMIZATION AND GAMES
UNIVERSITÉ PARIS–SACLAY



FINER APPROACHABILITY BOUNDS

Consider the approachability framework from the course and associated notation. Let $\mathcal{C} \subset \mathbb{R}^d$ be a nonempty closed convex cone satisfying Blackwell's condition, and α an associated oracle.

- 1) Prove that Blackwell's algorithm guarantees,

$$\min_{r \in \mathcal{C}} \left\| \sum_{t=1}^T r_t - r \right\|_2 \leq \sqrt{\sum_{t=1}^T \mathbb{1}_{\{r_t \notin \mathcal{C}\}} \|r_t\|_2^2}, \quad T \geq 1.$$

- 2) Can you prove that this bound is unimprovable (for Blackwell's algorithm, without further assumption)?
- 3) Can you prove a similar bound for the greedy Blackwell algorithm?
- 4) Can you generalize this result to other approachability algorithms?

