Evaluation
Online learning
links with optimization and games
Université Paris–Saclay

matrix exponential weights

Let $n \geqslant 1$ be an integer and $d = n^2$. We identify $\mathbb{R}^d$ with the space $\mathbb{R}^{n\times n}$ of square real matrices of size n. Denote $\mathscr{S}_+^n$ the set of real symmetric positive semidefinite matrices. Let $\mathscr{X} = \{X \in \mathscr{S}_+^n,\ \operatorname{Tr} X = 1\}$ and $h : \mathbb{R}^{n\times n} \to \mathbb{R} \cup \{+\infty\}$ defined as

$$h(X) = \begin{cases} \sum_{i=1}^n \lambda_i \log \lambda_i & \text{if } X \in \mathscr{X}\text{, where } \lambda_1, \dots, \lambda_n \text{ are the eigenvalues of } X \\ +\infty & \text{otherwise.} \end{cases}$$

1) Prove that $\mathscr{X}$ is a closed convex set.

2) Prove that h is well-defined.

3) Prove that h is a regularizer.

4) Is h strictly convex? strongly convex?

5) Compute $\min h$ and $\sup_{\mathscr{X}} h$. Is the supremum a maximum?

6) Give expressions for h^* and ∇h^*.

7) Define UMD sequences based on regularizer h and derive regret bounds.