Evaluation
Online learning
links with optimization and games
Université Paris–Saclay

yet another algorithm for online strongly convex optimization

Let $\mathcal{X}$ be a nonempty closed convex subset of $\mathbb{R}^d$, $\|\cdot\|$ be a norm in $\mathbb{R}^d$, $(K_t)_{t\geqslant 1}$ be a positive sequence and $(\ell_t)_{t\geqslant 1}$ convex losses on $\mathcal{X}$ such that for all $t \geqslant 1$, ℓ_t is K_t-strongly convex for $\|\cdot\|$. Let $(\rho_t)_{t\geqslant 1}$ be a sequence of regularizers on $\mathcal{X}$. For each $t \geqslant 1$, let $g_t \in \partial \ell_t(x_t)$ and consider

$$x_t = \operatorname*{arg\,min}_{x\in\mathcal{X}} \left\{ \sum_{s=1}^{t-1} \left(\langle g_s, x\rangle + \frac{K_s}{2} \|x - x_s\|^2 \right) + \rho_t(x) \right\}.$$

1) Prove that the above can be written as UMD iterates.

2) Derive regret bounds on $\sum_{t=1}^{T} (\ell_t(x_t) - \ell_t(x))$ (for $x \in \mathcal{X}$). *Hint: One possible approach is to consider:*

$$\tilde{\ell}_t(x) = \langle g_t, x\rangle + \frac{K_t}{2}\|x - x_t\|^2, \qquad x \in \mathbb{R}^d,\ t \geqslant 1,$$

and to remark that for all $t \geqslant 1$ and $x \in \mathcal{X}$,

$$\ell_t(x_t) - \ell_t(x) \leqslant \tilde{\ell}_t(x_t) - \tilde{\ell}_t(x).$$

3) In the case where $\|\cdot\| = \|\cdot\|_2$, derive regret bounds for

$$x_t = \operatorname*{arg\,min}_{x \in \mathscr{X}} \left\{ \sum_{s=1}^{t-1} \left(\langle g_s, x \rangle + \frac{K_s}{2} \|x - x_s\|^2 \right) \right\}, \quad t \geqslant 1.$$