

EVALUATION
ONLINE LEARNING
LINKS WITH OPTIMIZATION AND GAMES
UNIVERSITÉ PARIS–SACLAY



ONLINE LINEAR CLASSIFICATION WITH ABSOLUTE LOSS

Let $d \geq 1$, $\mathcal{X} \subset \mathbb{R}^d$ a nonempty closed convex set and $\mathcal{W} \subset \mathbb{R}^d$ a nonempty set such that

$$\forall x \in \mathcal{X}, \forall w \in \mathcal{W}, \quad |\langle w, x \rangle| \leq 1.$$

We consider the following online linear classification problem. At step $t \geq 0$,

- Nature chooses and reveals $w_t \in \mathcal{W}$,
 - the Decision Maker chooses $x_t \in \mathcal{X}$
 - Nature chooses $z_t \in \{-1, 1\}$
 - draw $\hat{z}_t = \begin{cases} 1 & \text{with probability } \frac{\langle w_t, x_t \rangle + 1}{2} \\ -1 & \text{with probability } \frac{1 - \langle w_t, x_t \rangle}{2} \end{cases}$,
 - Nature reveals z_t and the Decision Maker incurs loss $|\hat{z}_t - z_t|$.
- 1) By considering expectations, explain how the above problem can be reduced to a deterministic problem with convex loss functions $\ell_t(x) = |\langle w_t, x \rangle - z_t|$.

- 2) Propose at least two algorithms for this problem and derive corresponding regret guarantees.

