

EVALUATION
ONLINE LEARNING
LINKS WITH OPTIMIZATION AND GAMES
UNIVERSITÉ PARIS–SACLAY



ADAGRAD-FULL

Let $d \geq 1$ an integer, $\mathcal{X} \subset \mathbb{R}^d$ a nonempty closed convex set, $(u_t)_{t \geq 1}$ a sequence in \mathbb{R}^d , $x_1 \in \mathcal{X}$ and $\varepsilon, \gamma > 0$. We define

$$x_{t+1} = \arg \max_{x \in \mathcal{X}} \left\{ \langle \gamma u_t, x \rangle - \frac{1}{2} \|x - x_t\|_{A_t}^2 \right\}, \quad t \geq 1,$$

where

$$A_t = \varepsilon I_d + \sqrt{\sum_{s=1}^t u_s u_s^\top}, \quad t \geq 1,$$

where the square-root is to be understood component-wise.

- 1) Show that the above can be seen as UMD iterates.
- 2) Establish a general regret bound.
- 3) Deduce guarantees in the context of nonsmooth convex optimization and smooth convex optimization.

