Evaluation

Online learning

links with optimization and games

Université Paris–Saclay

solving games with alternating iterations

Let $m, n \geqslant 1$ be integers, $A \in \mathbb{R}^{m\times n}$, $\|\cdot\|_{(m)}$ and $\|\cdot\|_{(n)}$ norms on $\mathbb{R}^m$ and $\mathbb{R}^n$ respectively and denote $\|\cdot\|_{(m*)}$ and $\|\cdot\|_{(n*)}$ their respective dual norms, $h^{(m)}$ and $h^{(n)}$ regularizers on Δ_m and Δ_n, $((a_t, b_t, y_t, z_t))_{t\geqslant 0}$ a sequence in $\mathbb{R}^m \times \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^n$ and $K^{(m)}, K^{(n)}, \eta, \eta' > 0$. We assume that

- $((a_t, y_t))_{t\geqslant 0}$ is a sequence of strict UMD iterates associated with regularizer $h^{(m)}$ and dual increments $(\eta A(2b_t - b_{t-1}))_{t\geqslant 0}$ (with convention $b_{-1} = 0$),
- $((b_t, z_t))_{t\geqslant 0}$ is a sequence of strict UMD iterates associated with regularizer $h^{(n)}$ and dual increments $(\eta' A^\top a_{t+1})_{t\geqslant 0}$,
- $h^{(m)}$ is $K^{(m)}$-strongly convex for $\|\cdot\|_{(m)}$,
- $h^{(n)}$ is $K^{(n)}$-strongly convex for $\|\cdot\|_{(n)}$.

Let $T \geqslant 0$, $a \in \Delta_m$ and $b \in \Delta_n$.

1) Informally compare the above iterates with optimistic regret learning in two-player zero-sum games.

2) Recall the bound on

$$\sum_{t=0}^{T} \langle \eta A b_t, a - a_t \rangle .$$

3) Prove that

$$\sum_{t=0}^{T} \langle \eta' A^\top a_{t+1}, b - b_{t+1} \rangle \leqslant D_{h^{(n)}}(b, b_0;\ z_0) - \frac{K^{(n)}}{2} \sum_{t=0}^{T} \|b_{t+1} - b_t\|_{(m)}^2 .$$

4) Deduce a convergence guarantee in the context of solving the two-player zero-sum game associated with matrix A.

5) Express $(a_t)_{t\geqslant 0}$ and $(b_t)_{t\geqslant 0}$ in the special case of $h^{(m)}$ and $h^{(n)}$ being the entropic regularizers on Δ_m and Δ_n respectively. Write the corresponding convergence guarantee.

6) Conduct numerical experiments to compare the performance of the algorithm from the last question with the optimistic exponential weights algorithm.

7) Bonus. — Include in the numerical experiments other algorithms e.g. RM, RM+, regular exponential weights algorithms.