

EVALUATION
ONLINE LEARNING
LINKS WITH OPTIMIZATION AND GAMES
UNIVERSITÉ PARIS–SACLAY



ALTERNATIVES TO FTRL AND FTL

In the context of online convex optimization with differentiable losses, let H be a mirror map compatible with \mathcal{X} , $x_0 \in \mathcal{X} \cap \text{int dom } H$ and consider

$$x_{t+1} = \arg \min_{x \in \mathcal{X}} \left\{ \langle \nabla \ell_t(x_t), x \rangle + \sum_{s=0}^t D_{\ell_s}(x, x_t) + D_H(x, x_t) \right\}.$$

- 1) Prove that the above can be interpreted as UMD iterates.
- 2) Derive a regret bound (on $\sum_{t=0}^T (\ell_t(x_t) - \ell_t(x))$) and compare with the corresponding regret bound for FTRL.
- 3) Generalize with a sequence of time-dependent mirror maps $(H_t)_{t \geq 0}$ and derive similar results as for FTRL.
- 4) In the case of differentiable strongly convex losses, consider

$$x_{t+1} = \arg \min_{x \in \mathcal{X}} \left\{ \langle \nabla \ell_t(x_t), x \rangle + \sum_{s=0}^t D_{\ell_s}(x, x_t) \right\},$$

and derive regret guarantees.

