# Evaluation
# Online learning
## links with optimization and games
## Université Paris–Saclay

### asynchronous fixed point iterations

Let $d \geqslant 1$, $\mathscr{X} \subset \mathbb{R}^d$ a nonempty closed convex set, and $F : \mathscr{X} \to \mathscr{X}$ a map. For $1 \leqslant i \leqslant d$, denote $F^{|i} : \mathscr{X} \to \mathscr{X}$ the map defined as

$$F^{|i}(x)_j = \begin{cases} F(x)_j & \text{if } j = i \\ x_j & \text{otherwise,} \end{cases} \qquad 1 \leqslant j \leqslant d.$$

1) Prove that $x \in \mathscr{X}$ is a fixed point of $F$ if, and only if, for all $1 \leqslant i \leqslant d$, it is a fixed point of $F^{|i}$.

2) Prove that if $F$ is nonexpansive, then $F^{|i}$ is nonexpansive for all $1 \leqslant i \leqslant d$.

3) Let $L > 0$. Prove that if $\frac{1}{L}F + (1 - \frac{1}{L})I$ is nonexpansive, then $\frac{1}{2}(I - F^{|i})$ is L-co-coercive for all $1 \leqslant i \leqslant d$.

4) Assume that $F$ is nonexpansive and let $x_*$ be a fixed point of $F$. Let $(i_t)_{t \geqslant 0}$ be a sequence in $\{1, \dots, d\}$, $(\gamma_t)_{t \geqslant 0}$ a sequence in $(0, 1)$, $x_0 \in \mathscr{X}$, and for $t \geqslant 0$,

$$x_{t+1} = (1 - \gamma_t)x_t + \gamma_t F^{|i_t}(x_t).$$

The above is an asynchronous counterpart of the Krasnoselskii-Mann iteration. Let $T \geqslant 0$. Establish a guarantee on

$$\min_{0 \leqslant t \leqslant T} \|F(x_t) - x_t\|_2 .$$

5) Let $L > 0$ and assume that $\frac{1}{L}F + (1 - \frac{1}{L})I$ is nonexpansive. Define and analyze an asynchronous fixed point iteration for F, based on AdaGrad-Norm (resp. AdaGrad-Diagonal).