EVALUATION

ONLINE LEARNING

LINKS WITH OPTIMIZATION AND GAMES

UNIVERSITÉ PARIS–SACLAY

EXPONENTIAL WEIGHTS WITH STEP-SIZES

Let $(u_t)_{t\geqslant 0}$ be a sequence in $\mathbb{R}^d$ and $(\gamma_t)_{t\geqslant 0}$ a nonincreasing sequence in $\mathbb{R}^d$. Consider:

$$x_t = \left(\frac{\exp\left(\left(\sum_{s=0}^{t-1} \gamma_s u_{s,i}\right)\right)}{\sum_{j=1}^{d} \exp\left(\sum_{s=0}^{t-1} \gamma_s u_{s,j}\right)} \right)_{1\leqslant i\leqslant d}. \tag{1}$$

1) Let $T \geqslant 0$ and $x \in \Delta_d$. Derive a general bound on $\sum_{t=0}^{T} \langle u_t, x - x_t\rangle$.

2) Derive a regret bound in the case where there exists $L > 0$ such that $\|u_t\|_\infty \leqslant L$ for all $t \geqslant 0$.

3) In the multi-armed bandit problem, consider the variant of EXP3 based on (1), and derive a guarantee with similar assumptions as for EXP3 in the course.