EVALUATION

ONLINE LEARNING

LINKS WITH OPTIMIZATION AND GAMES

UNIVERSITÉ PARIS–SACLAY

FINER APPROACHABILITY BOUNDS

Consider the approchability framework from the course and associated notation. Let $\mathscr{C} \subset \mathbb{R}^d$ be a nonempty closed convex cone satisfying Blackwell's condition, and α an associated oracle.

1) Prove that Blackwell's algorithm guarantees,

$$\min_{r \in \mathscr{C}} \left\| \sum_{t=1}^{T} r_t - r \right\|_2 \leqslant \sqrt{\sum_{t=1}^{T} \mathbb{1}_{\{r_t \notin \mathscr{C}\}} \|r_t\|_2^2}, \quad T \geqslant 1.$$

2) Can you prove that this bound is unimprovable (for Blackwell's algorithm, without further assumption)?

3) Can you prove a similar bound for the greedy Blackwell algorithm?

4) Can you generalize this result to other approachability algorithms?