Evaluation
Online learning
links with optimization and games
Université Paris–Saclay

solving games with line-search

Let $d \geqslant 1$, $\mathcal{X} \subset \mathbb{R}^d$ a nonempty closed convex set, h a regularizer on $\mathcal{X}$, $G : \mathcal{X} \to \mathbb{R}^d$ a monotone operator and $(\gamma_t)_{t \geqslant 0}$ a positive sequence.

We consider UMP iterates with time-dependent step-sizes. Let $((x_t, w_t, y_t, z_t))_{t \geqslant 0}$ be such that $((x_t, y_t))_{t \geqslant 0}$ is sequence of strict UMD iterates associated with regularizer h and dual iterates $(-\gamma_t G(w_t))$ and for $t \geqslant 0$,

(i) $z_t \in \partial h(x_t)$,

(ii) $\forall x \in \mathcal{X},\ \langle z_t - y_t, x - x_t \rangle \geqslant 0$,

(iii) $w_t = \nabla h^*(z_t - \gamma_t G(x_t))$.

1) Prove that if

$$\forall t \geqslant 0, \quad \gamma_t \langle G(w_t), x_{t+1} - w_t \rangle \leqslant D_h(x_{t+1}, x_t;\ y_t), \tag{1}$$

then for all $T \geqslant 0$,

$$\forall x \in \operatorname{dom} h, \quad \left\langle G(x), \bar{w}_T^{(\gamma)} - x \right\rangle \leqslant \frac{D_h(x, x_0;\ y_0)}{\sum_{t=0}^{T} \gamma_t},$$

where $\bar{w}_{\mathrm{T}}^{(\gamma)} = \left(\sum_{t=0}^{\mathrm{T}} \gamma_t\right)^{-1} \sum_{t=0}^{\mathrm{T}} \gamma_t w_t$.

Hint. — Adapt the proof from the course.

The above guarantee encourages to choose values for γ_t that are large while satisfying condition (1).

2) Propose an algorithmic scheme ("line-search") for choosing a value for γ_t in the logarithmic scale $\left\{\sqrt{2}^k\right\}_{k\in\mathbb{Z}}$ so that condition (1) is satisfied.

We now consider the following regularizer. Let $m, n \geqslant 1$ and let $h : \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ be defined as

$$h(a, b) = h_{\mathrm{ent}}(a) + h_{\mathrm{ent}}(b), \qquad (a, b) \in \mathbb{R}^m \times \mathbb{R}^n,$$

where h_{ent} denotes both the entropic regularizer on Δ_m and on Δ_n.

3) Prove that h is a regularizer on $\Delta_m \times \Delta_n$.

4) Prove that $(x_t)_{t\geqslant 0}$ and $(w_t)_{t\geqslant 0}$ are then uniquely determined.

5) Give an explicit expression for $(x_t)_{t\geqslant 0}$ and $(y_t)_{t\geqslant 0}$ in the special case of solving a two-player zero-sum game, and write the corresponding guarantee.

6) Perform numerical experiments in the context of solving two-player zero-sum games to compare the performance of the above algorithm (with line-search) with the same algorithm with no line-search, the optimistic exponential weights algorithm, RM and RM+.