# Evaluation
# Online learning
## links with optimization and games
Université Paris–Saclay

### generalized regularizers

We aim at extending the foundations of UMD theory to weaker assumptions for the regularizer. Let $d \geqslant 1$ and $\mathscr{X}$ be a nonempty closed convex subset of $\mathbb{R}^d$.

Définition. — A function $h : \mathbb{R}^d \to \mathbb{R} \cup \{+\infty\}$ is a *generalized regularizer* on $\mathscr{X}$ if is convex, lower semicontinuous and if $\operatorname{cl} \operatorname{dom} h = \mathscr{X}$.

Remarque. — Compared to the definition of usual regularizers, strong convexity is not required and $\operatorname{dom} h^*$ may be a strict subset of $\mathbb{R}^d$.

Définition. — Let $h$ be a generalized regularizer on $\mathscr{X}$ and $(u_t)_{t \geqslant 0}$ a sequence in $\mathbb{R}^d$. A sequence $((x_t, y_t))_{t \geqslant 0}$ in $\mathbb{R}^d \times \mathbb{R}^d$ is an associated sequence of UMD associated if for all $t \geqslant 0$,

(i) $y_t \in \partial h(x_t)$

(ii) $x_{t+1} \in \partial h^*(y_t + u_t)$.

These are *strict UMD iterates* if moreover for all $t \geqslant 0$,

$$\forall x \in \mathscr{X}, \quad \langle y_t + u_t - y_{t+1}, x - x_{t+1} \rangle \leqslant 0.$$

Remarque. — Condition $x_{t+1} \in \partial h^*(y_t + u_t)$ implies in particular that $y_t + u_t \in \operatorname{dom} h^*$.

We also define UMD iterates with respect to time-dependent generalized regularizers.

Définition. — Let $(h_t)_{t \in \frac{1}{2}\mathbb{N}}$ be a sequence of generalized regularizers on $\mathcal{X}$ and $(u_t)_{t \geqslant 0}$ a sequence in $\mathbb{R}^d$. A sequence $((x_t, y_t))_{t \geqslant 0}$ in $\mathbb{R}^d \times \mathbb{R}^d$ is an associated sequence of UMD iterates if for all $t \in \mathbb{N}$,

(i) $y_t \in \partial h_t(x_t)$,

(ii) $x_{t+1} = \partial h^*_{t+1/2}(y_t + u_t)$.

*How can you extend/adapt the results from Chapter 2 to generalized regularizers?*