Evaluation

Online learning

links with optimization and games

Université Paris–Saclay

a simple market game

1) Let $\mathscr{T}$ be a treeplex as defined in the course. For $\theta \in \Theta$ and $a \in \mathscr{A}_\theta$, denote $\mathbb{1}_{[\theta,a]}$ the element in $\Delta(\mathscr{A}_\theta)$ that is the Dirac at a. Prove that $\mathscr{T}$ is the set of convex combinations of the points from the following set

$$\mathscr{T}_0 = \left\{ x \in \mathbb{R}_+^\Sigma,\ \forall \theta \in \Theta,\ \exists a \in \mathscr{A}_a,\ x_{[\theta]} = x_{[p(\theta)]} \mathbb{1}_{[\theta,a]}. \right\}.$$

Therefore, an useful consequence is that for all $v \in \mathbb{R}^\Sigma$,

$$\max_{x \in \mathscr{T}} \langle v, x \rangle = \max_{x \in \mathscr{T}_0} \langle v, x \rangle .$$

We consider the following two-player zero-sum game. Let $p, q \in [0,1]$ be given. Where Player 1 is a potential new-comer to a market, while Player 2 is already present. The market can either be in a *high demand* state (with probability p) or in a *low demand* state (with probability $1 - p$). Only Player 1 observes the state of the market. Then, Player 1 chooses to either *enter* the market or to *stay out*. If he stays out, both players get payoff 0. If he *enters*, he incurs either a *low cost* of 0 for entering the market (with probability q) or a *high cost* of 2 (with probability $1 - q$). This *entering cost* will be taken into account in the payoff. Player 1 observes whether he incurs *high* or *low* entering cost, Player 2 does not observe it. Then, Player 2 either chooses to *fight* or to *accomodate*. If the demand is *high*, Player 1 gets a payoff equal to: *minus* the entering cost *plus* 5 if Player 2 *fights* or 8 if Player 2 *accomodates*. If the demand is *low*, Player 1 gets a payoff equal to: *minus* the entering cost *plus* 1 if Player 2 *fights* or 3 if Player 2 *accomodates*.

2) Formally write the game as an extensive-form game.

3) Solve the game using the CFR and CFR+ algorithms. Plot the convergence. Describe the obtained approximate solutions. Try different values for p and q.