EVALUATION

ONLINE LEARNING

LINKS WITH OPTIMIZATION AND GAMES

UNIVERSITÉ PARIS–SACLAY

OPTIMISTIC ITERATIONS WITH ARBITRARY NORMS FOR SOLVING GAMES

The goal of this project is to extend the guarantee of the optimistic exponential weights algorithm in the context of solving two-player zero-sum games.

Let $m, n \geqslant 1$ be integers, $A \in \mathbb{R}^{m\times n}$, $\|\cdot\|_{(m)}$ and $\|\cdot\|_{(n)}$ norms on $\mathbb{R}^m$ and $\mathbb{R}^n$ respectively and denote $\|\cdot\|_{(m*)}$ and $\|\cdot\|_{(n*)}$ their respective dual norms, $h^{(m)}$ and $h^{(n)}$ regularizers on Δ_m and Δ_n, $((a_t, b_t, y_t, z_t))_{t\geqslant 0}$ a sequence in $\mathbb{R}^m \times \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^n$ and $K^{(m)}$, $K^{(n)}$, $\eta, \eta' > 0$. We assume that

- $((a_t, y_t))_{t\geqslant 0}$ is a sequence of strict UMD iterates associated with regularizer $h^{(m)}$ and dual increments $(\eta A(2b_t - b_{t-1}))_{t\geqslant 0}$ (with convention $b_{-1} = 0$),
- $((b_t, z_t))_{t\geqslant 0}$ is a sequence of strict UMD iterates associated with regularizer $h^{(n)}$ and dual increments $(\eta' A^\top(2a_t - a_{t-1}))_{t\geqslant 0}$ (with convention $a_{-1} = 0$),
- $h^{(m)}$ is $K^{(m)}$-strongly convex for $\|\cdot\|_{(m)}$,
- $h^{(n)}$ is $K^{(n)}$-strongly convex for $\|\cdot\|_{(n)}$.

Question. — Let $T \geqslant 0$, $\bar{a}_T = \frac{1}{T+1} \sum_{t=0}^{T} a_t$ and $\bar{b}_T = \frac{1}{T+1} \sum_{t=0}^{T} b_t$. Derive a guarantee on

$$\delta_A(\bar{a}_T, \bar{b}_T).$$