# Evaluation
# Online Learning
## links with optimization and games
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### perceptron

Let $d \geqslant 1$ and consider the following online linear classification problem. At step $t \geqslant 0$,

- Nature chooses and reveals $w_t \in \mathscr{W}$,
- the Decision Maker chooses $x_t \in \mathbb{R}^d$, and denote[1] $\hat{z}_t = \operatorname{sign} \langle w_t, x_t \rangle$,
- Nature chooses and reveals $z_t \in \{-1, 1\}$,
- the Decision Maker incurs loss $\mathbb{1}\{\hat{z}_t \neq z_t\}$.

Consider the following algorithm called *Perceptron*. Let $x_0 = 0$ and

$$x_{t+1} = x_t + \mathbb{1}\{\hat{z}_t \neq z_t\} z_t w_t, \quad t \geqslant 0.$$

1) Prove that the above are UMD iterates.

2) Establish an upper bound on the following quantity

$$\sum_{t=0}^{T} \mathbb{1}\{\hat{z}_t \neq z_t\} - \sum_{t=0}^{T} \max(1 - \langle w_t, x \rangle z_t, 0), \quad x \in \mathbb{R}^d, \ T \geqslant 0.$$

[1] $\operatorname{sign}(a)$ is the value in $\{-1, 0, 1\}$ with the same strict sign as $a$.