EVALUATION

ONLINE LEARNING

LINKS WITH OPTIMIZATION AND GAMES

UNIVERSITÉ PARIS–SACLAY

CONVEX OPTIMIZATION WITH RELATIVE SMOOTHNESS

Let $d \geqslant 1$ an integer, $\mathcal{X} \subset \mathbb{R}^d$ a closed convex set, $f : \mathbb{R}^d \to \mathbb{R}$ a convex and differentiable function that admits a minimizer $x_* \in \mathcal{X}$ on $\mathcal{X}$:

$$f(x_*) = \min_{x \in \mathcal{X}} f(x).$$

Let $\mathrm{L} > 0$ and $h : \mathbb{R}^d \to \mathbb{R} \cup \{+\infty\}$ a regularizer such that $\mathrm{L}h - f$ is convex.

1) Prove that for all $x, x' \in \mathbb{R}^d$ and $y \in \partial h(x)$,

$$f(x') \leqslant f(x) + \langle \nabla f(x), x' - x \rangle + \mathrm{L}D_h(x', x;\ y).$$

2) Extend Section 6.2 by defining and analyzing UMD-based algorithms in the above context.

3) BONUS. — Extend Nesterov's acceleration from Section 6.3.