Evaluation
Online learning
links with optimization and games
Université Paris–Saclay

iterates based on squared Mahalanobis norms

Let $d \geqslant 1$. Recall that for a symmetric positive definite matrix A of size $d \times d$, the associated Mahalanobis norm is defined as

$$\|x\|_A = \sqrt{\langle x, Ax \rangle}, \qquad x \in \mathbb{R}^d.$$

Let $\mathscr{X} \subset \mathbb{R}^d$ be a nonempty closed convex set, $(A_t)_{t \geqslant 0}$ a sequence of symmetric positive definite matrices of size $d \times d$, and $(u_t)_{t \geqslant 0}$ a sequence in $\mathbb{R}^d$. For each iterates definition below, prove that they are UMD iterates and derive bounds on the regret $\sum_{t=0}^{T} \langle u_t, x - x_t \rangle$ for $T \geqslant 0$ and $x \in \mathscr{X}$.

1) Let $x_0 \in \mathscr{X}$ and

$$x_{t+1} = \operatorname*{arg\,min}_{x \in \mathscr{X}} \left\| (x_t + A_t^{-1} u_t) - x \right\|_{A_t}, \qquad t \geqslant 0.$$

2) Let $x_0 \in \mathscr{X}$ and

$$x_{t+1} = \operatorname*{arg\,min}_{x \in \mathscr{X}} \left\{ - \langle A_t x_t + u_t, x \rangle - \frac{1}{2} x^\top A_{t+1} x \right\}, \qquad t \geqslant 0.$$

3) Let $y_0 \in \mathbb{R}^d$ and

$$x_t = \arg\min_{x \in \mathcal{X}} \left\{ - \left\langle y_0 + \sum_{s=0}^{t-1} u_s, x \right\rangle + \frac{1}{2} \langle x, \mathrm{A}_t x \rangle \right\}, \quad t \geqslant 0.$$