ONLINE LEARNING
LINKS WITH OPTIMIZATION AND GAMES
UNIVERSITÉ PARIS–SACLAY

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YET ANOTHER ALGORITHM FOR ONLINE STRONGLY CONVEX OPTIMIZATION

Let $\|\cdot\|$ be a norm, $(K_t)_{t\geqslant 0}$ be a positive sequence and $(\ell_t)_{t\geqslant 0}$ convex losses on $\mathscr{X}$ such that for all $t \geqslant 0$, ℓ_t is K_t-strongly convex for $\|\cdot\|$. Let $(\rho_t)_{t\geqslant 0}$ be a sequence of regularizers on $\mathscr{X}$. For each $t \geqslant 0$, let $g_t \in \partial\ell_t(x_t)$ and consider

$$x_t = \arg\min_{x\in\mathscr{X}} \left\{ \sum_{s=0}^{t-1}\left(\langle g_s, x\rangle + \frac{K_s}{2}\|x - x_s\|^2\right) + \rho_t(x) \right\}.$$

1) Prove that the above can be written as UMD iterates.

2) Derive regret bounds on $\sum_{t=0}^{T}\left(\ell_t(x_t) - \ell_t(x)\right)$ (for $x \in \mathscr{X}$). *Hint: One possible approach is to consider:*

$$\tilde{\ell}_t(x) = \langle g_t, x\rangle + \frac{K_t}{2}\|x - x_t\|^2, \quad x \in \mathbb{R}^d,\ t \geqslant 0,$$

and to remark that for all $t \geqslant 0$ and $x \in \mathscr{X}$,

$$\ell_t(x_t) - \ell_t(x) \leqslant \tilde{\ell}_t(x_t) - \tilde{\ell}_t(x).$$

3) In the case where $\| \cdot \| = \| \cdot \|_2$, derive regret bounds for

$$x_t = \arg\min_{x \in \mathcal{X}} \left\{ \sum_{s=0}^{t-1} \left(\langle g_s, x \rangle + \frac{K_s}{2} \|x - x_s\|^2 \right) \right\}, \quad t \geqslant 0.$$