Evaluation
Online learning
links with optimization and games
Université Paris–Saclay

UMD-based extension of AdaGrad-Norm and application to games

Let $d \geqslant 1$, $\mathcal{X} \subset \mathbb{R}^d$ a nonempty closed convex set, $K > 0$, $\|\cdot\|$ a norm on $\mathbb{R}^d$, h a regularizer on $\mathcal{X}$ that is K-strongly convex for $\|\cdot\|$.

Let $\gamma > 0$. For $(u_t)_{t\geqslant 0}$ a sequence in $\mathbb{R}^d$, let $((x_t, y_t))_{t\geqslant 0}$ be a sequence of strict UMD iterates associated with regularizer h and dual increments $(\gamma_t u_t)_{t\geqslant 0}$, where

$$\gamma_t = \frac{\gamma}{\sqrt{\sum_{s=0}^{t} \|u_s\|_*^2}}, \qquad t \geqslant 0,$$

with convention $0/0 = 0$.

1) For $x \in \operatorname{dom} h$ and $T \geqslant 0$, derive a guarante on the regret

$$\sum_{t=0}^{T} \langle u_t, x - x_t \rangle .$$

2) a) In the special cases of dual averaging (with a constant regularizer and dual increments $(\gamma_t u_t)_{t\geqslant 0}$) and online mirror descent (with a constant mirror map and dual increments $(\gamma_t u_t)_{t\geqslant 0}$), derive corresponding algorithms and guarantees.

b) Write corollaries for dual averaging with Euclidean regularizer and mirror descent with Euclidean mirror map.

c) For the entropic regularizer on the simplex, derive the corresponding algorithm and guarantee.

3) Apply to regret learning for finite two-player zero-sum games and derive guarantees. Perform numerical experiments to compare the convergence of the above algorithms (Euclidean DA, Euclidean MD, entropic regularizer) with RM, RM+ and classical exponential weights.

4) Bonus. — Apply to various optimization problems and derive guarantees.